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HYDRODYNAMICS OF DISPERSED-ANNULAR GAS–LIQUID STREAMS IN BUNDLES OF RODS

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On the basis of concepts involving dispersed-annular gas–liquid streams in circular pipes [1, 2], within the framework of the cell model of the stream, we have constructed a one-dimensional stationary hydrodynamic model of the flow of a gas–liquid mixture in a dispersed-annular regime in channels with bundles of rods. We have analyzed the mass and force interactions between the components of the dispersed-film stream within the cells and between the cells. We obtained satisfactory agreement with the numerical and experimental data published in the literature with respect to hydraulic resistance in channels with bundles of heated rods having various geometries. We have shown the variation of the main hydrodynamic characteristics of the steam–water dispersed-annular stream over the cross section and along the length of the channel to be functions of the operational parameters of the mixture.

The development of calculation methods for the hydrodynamics and the heat-transfer crisis in channels with bundles of rods can follow two different lines: in the first place, it can proceed on the basis of methods using averaged parameters of the coolant in the channel; in the second place, it can be based on cell models taking account of convective and turbulent mixing of the phases between the cells. The first line has been developed to a considerable extent in the processing of direct experimental data on hydraulic resistance and resistance crises [3].

The second line is more general and flexible than the first, but it is, naturally, more cumbersome. The investigation of the hydrodynamics and the heat-transfer crisis in channels with bundles of rods, using cell representations, is usually carried out within the framework of a homogeneous model of a gas–liquid stream. This approach has been developed most coherently in [4].

At a volumetric concentration of the gaseous phase higher than 0.6-0.8 [5] the steam–water stream in a channel with bundles of rods moves in a dispersed-film regime of flow. The homogeneous model does not take account of the most characteristic features of such a flow,

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which is characterized by the presence of liquid films on the rods and the jacket and of liquid drops in the gaseous medium in the space between the rods. The disappearance of the liquid film on one of the heated rods leads to the occurrence of a heat-transfer crisis in the channel. Within the framework of the homogeneous model, a detailed description of this process is impossible, and therefore when this model is used for determining the conditions of occurrence of the heat-transfer crisis, we must use experimental data on the heat-transfer crisis that have been obtained in circular pipes. This approach is of limited validity in the determination of a heat-transfer crisis in channels with bundles of rods. By taking account of the dispersed-film structure of the gas—liquid stream, we can describe in more detail the flow of the gas—liquid mixture and determine in a natural way the conditions for the occurrence of the heat-transfer crisis that are connected with the drying-up of the liquid film on the walls.

In taking account of the dispersed-film structure of the flow, it is convenient to distinguish the cells around each rod. Then within an individual cell, as is done in pipes and annular channels, we can distinguish three components of the mixture: the gaseous phase, the liquid film, and the drops. As the boundaries between the cells in this case, we may take the curves with zero tangential stress [6]. However, there are considerable difficulties in the exact determination of these curves when there are transverse streams of steam and drops, and therefore, on the basis of symmetry, we can take as such curves the segments of curves passing between the rods at an equal distance from each (Fig. 1). An attempt to take account of the dispersed-film structure of the flow of the mixture was made in [7]. It is limited in nature because it used very rough approximations to the functions for the intensities of moisture exchange and force interaction within a cell.

In recent years we have carried out a number of experimental investigations over a wide range of variation of regime parameters [8-10] to determine the intensities of the force and mass interaction between the components of the mixture in a steam—water dispersed-annular stream in a circular pipe. The results of these investigations make it possible to construct in the most coherent manner a closed hydrodynamic model of the flow of the gas—liquid mixture in a dispersed-film regime in channels with bundles of rods.

1. Formulation of the Problem; Fundamental Assumptions. We consider a stationary gas—liquid dispersed-film stream in a vertical channel with N rods. Figure 1 shows a cross-section of this on N cells. Suppose that in each cell there is a liquid film flowing along the rod, and in the space between the surface of the liquid film and the boundary of the cell there moves a gaseous phase with drops of liquid. In the peripheral cells which are adjacent to the unheated jacket, the liquid film also moves along the surface of the jacket. We assume that we can disregard the present gradient in the transverse direction in comparison with the longitudinal pressure gradient in the channel, i.e., we assume that the pressure at any cross section of the channel is uniform in all the cells. At mixture flow rates much smaller than the critical flow rates, we can assume that the mixture is in thermodynamic equilibrium (the temperatures of the phases are equal to the saturation temperature) and the velocities of the drops and the gaseous phase are equal to each other. It can also be shown that when the mixture is in a turbulent flow regime, the effect of the nonuniformity in the distribution of the concentration of drops, of the velocities of the gaseous phase, and of the drops over the cross section of the cell is slight. For circular pipes the validity of this assertion was shown in [1]. Lastly, we shall assume that the parameters of the liquid film are uniform over the perimeter of the rod. In what follows, we shall show that the most essential characteristic of the liquid film is the irrigation density, i.e., the flow rate of the liquid film adjacent to a unit length of rod perimeter. The uniformity in the distribution of this parameter with respect to the perimeter of the rod under various conditions of external gas flow past it has been shown experimentally in [11]. With the above assumptions, we can describe a dispersed-film stream in each cell within the framework of a one-dimensional model of dispersed-annular streams in pipes [1, 2].

2. Fundamental Equations of Conservation. We shall denote the parameters of the mixtures relating to the k-th cell by a superscript letter k, and the parameters relating to the gaseous phase, the liquid film, and the drops by subscript numerals 1, 2, and 3, respectively. The equation of conservation of mass in each cell for the three components of the mixture has the form

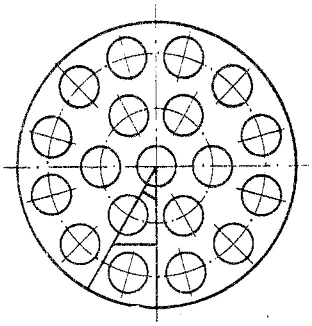

Fig. 1

$$\frac{d}{dz}\left(\rho_1^0\alpha_1^k v_1^k\right)=\frac{1}{S^k}\sum_{l=1}^{N} I^{l,k}\left(\frac{\alpha_1^l+\alpha_1^k}{2}\right)\rho_1^0+j_{21}^k+j_{31}^k-j_{12}^k-j_{13}^k, \tag{2.1}$$

$$\frac{d}{dz}\left(\rho_2^0\alpha_2^k v_2^k\right)=j_{12}^k+j_{32}^k-j_{23}^k-j_{21}^k,$$

$$\frac{d}{dz}\left(\rho_3^0\alpha_3^k v_3^k\right)=\frac{1}{S^k}\sum_{l=1}^{N} I^{l,k}\frac{\alpha_3^l+\alpha_3^k}{2}\rho_2^0 k_z+j_{23}^k+j_{13}^k-j_{31}^k-j_{32}^k+\frac{1}{S^k}\sum_{l=1}^{N} k_t^{l,k}k_z\rho_3^0\alpha_3^{l,k},$$

where ρ_1^0, $\rho_2^0\equiv\rho_3^0$ are the true densities of the vapor and the liquid; α_j^k is the volumetric concentration of the j-th component of the mixture (j = 1, 2, 3) in each cell; v_j^k is the velocity of the j-th component of the mixture (j = 1, 2, 3); S^k is the area of the k-th cell; $I^{l,k}$ is the convective stream of the gas–drop mixture from the l-th cell into the k-th cell, per unit length of the channel ($I^{l,k}=0$ if $l = k$ or l and k correspond to cells which have no common boundary); $j_{m,n}^k$ (m, n = 1, 2, 3; m ≠ n) is the intensity of mass exchange between the m-th and n-th components of the mixture in each cell, i.e., the amount of the m-th component that has passed into the n-th in a unit volume of each cell per unit time; $k_t^{l,k}$ is the coefficient of turbulent exchange of the gas–drop mixture between the l-th and k-th cells ($k_t^{l,k}=k_t^{k,l}$, $k_t^{l,k}=0$ if l and k correspond to cells which have no common boundary); $\alpha_3^{l,k}=\alpha_3^l/(\alpha_1^l+\alpha_3^l)-\alpha_3^k(\alpha_3^k/(\alpha_1^k+\alpha_3^k)$ is the difference between the volumetric concentrations of the drops in the gas–drop mixture in the l-th and k-th adjacent cells; k_z is the coefficient of slippage of the drops with respect to the gaseous phase in the transverse direction, equal to the ratio of the transverse velocity of the drops to the transverse velocity of the gaseous phase.

In the derivation of Eq. (2.1) it was assumed that the concentration of drops in the gas–drop mixture varies smoothly between two adjacent cells; then it can be assumed that the concentration of drops in the mixture flowing from the k-th cell into the l-th cell is equal to $(\alpha_3^k+\alpha_3^l)/2$, where we take account of the fact that the liquid film does not participate in the intercell exchange.

The equations of conservation of momentum for a single-velocity gas–drop core and a liquid film in the k-th cell as projected onto the longitudinal axis of the channel have the form

$$\begin{gathered}\left(\rho_1^0\alpha_1^k+\rho_2^0\alpha_3^k\right)v_1^k\frac{dv_1^k}{dz}=-\left(\alpha_1^k+\alpha_3^k\right)\frac{dp}{dz}-\frac{f_{12}}{S^k}-\left(\alpha_1^k\rho_1^0+\alpha_2^k\rho_2^0\right)g+\\ +\frac{1}{S^k}\sum_{l=1}^{N}I^{l,k}\left(\frac{v_1^l-v_1^k}{2}\right)\left(\frac{\alpha_1^l+\alpha_1^k}{2}\rho_1^0+k_z\frac{\alpha_3^l+\alpha_3^k}{2}\rho_2^0\right)+\frac{1}{S^k}\sum_{l=1}^{N}k_t^{l,k}\left(v_1^l-v_1^k\right)\times\\ \times\left[\rho_1^0\left(\frac{\alpha_1^k}{\alpha_1^k+\alpha_3^k}+\frac{\alpha_1^l}{\alpha_1^l+\alpha_3^l}\right)\frac{1}{2}+k_z\rho_2^0\left(\frac{\alpha_3^k}{\alpha_1^k+\alpha_3^k}+\frac{\alpha_3^l}{\alpha_1^l+\alpha_3^l}\right)\frac{1}{2}\right]+\\ +\left(j_{21}^k+j_{23}^k-j_{12}^k-j_{32}^k\right)\left(v_2^{\prime k}-v_1^k\right),\quad \rho_2^0\alpha_2^k v_2^k\frac{d}{dz}v_2^k=-\alpha_2^k\frac{dp}{dz}+\frac{f_w^k}{S^k}+\frac{f_{12}^k}{S^k}-\\ -\alpha_2^k\rho_2^0 g+\left(j_{12}^k+j_{32}^k-j_{21}^k-j_{23}^k\right)\left(v_1^k-v_2^{\prime k}\right),\end{gathered} \tag{2.2}$$

where p is the pressure in the mixture; f_w^k, f_{12}^k are, respectively, the frictional forces between the film and the rod and between the film and the core of the stream, per unit length of the channel; g is the acceleration of the free fall; $v_2^!$ is the velocity of the liquid at the surface of the film. The last terms on the right sides of the equations in (2.2) determine the variation of the momentum of the gas–drop core and the liquid film as a result of the intracell mass exchange, and in the first approximation, also as a result of the intercell mass exchange in the gas–drop core of the stream.

In deriving Eqs. (2.2) we took account of the fact that the variation of momentum for the gas–drop core and for the liquid film may be different because of the flow of mass from the drop core into the film and vice versa. Furthermore, we assumed that the velocities of the gaseous phase and the drops vary smoothly between two adjacent cells. Then we may assume that the longitudinal velocity of the gas–drop mixture passing from the k-th cell into the l-th cell is equal to $(v_1^k + v_1^l)/2$. For a thermodynamic-equilibrium gas–liquid stream, if we disregard the work required to compress the phases, the equations for the influx of heat for each component of the mixture can be written in the form

$$j_{21}^k = \left(-\rho_2^0\alpha_2^k v_2^k C_1 \frac{dT_s}{dp}\frac{dp}{dz} + Q^k\right)\Big/ l,$$
$$j_{31}^k = v_1^k \frac{dT_s}{dp}\left(\rho_1^0\alpha_1^k C_1 + \rho_2^0\alpha_3^k C_2\right)\frac{dp}{dz}\frac{1}{l}, \tag{2.3}$$

where $C_j(p)$ is the heat capacity of the j-th component of the mixture; $T_s(p)$ is the saturation temperature; $l(p)$ is the latent heat of vaporization; Q^k is the external heat generated in each cell. In deriving (2.3), we assumed that all the heat generated in the rod passes into the liquid film. In the heated channels there is no condensation of the vapor; then $j_{12}^k = j_{13}^k = 0$ (k = 1, 2, ..., N).

It should be noted that in the cells adjacent to the jacket, in addition to the three components of the mixture (the gaseous phase, the liquid film on the rod, and the drops), there is another component: the liquid film on the outer unheated jacket. This component is characterized by its volumetric concentration and its velocity in the cell. The equations for describing it are analogous to the equations for the liquid film on the rod and are distinguished only by the absence of a term taking account of the heat influx to the liquid film on the external (cold) boundary.

Equations (2.1)-(2.3) include the unknown convective streams of the volumes of the gas–drop mixture between adjacent cells, $I^{l,k}$, which are determined from the condition of constant pressure in the cross section of the channel. Indeed, from (2.1)-(2.3) and the condition of incompressibility of the liquid, $\rho_2^0 \equiv \rho_3^0$ = const, we can obtain the following equation describing the variation of the volumetric concentration of the j-th component (j = 1, 2, 3) of the mixture in the k-th cell:

$$\frac{d\alpha_1^k}{dz} = \frac{A_{11}^k}{\rho_1^0 v_1^k} - \frac{\alpha_1^k}{\rho_1^0}\frac{d\rho_1^0}{dp}\frac{dp}{dz} - \frac{\alpha_1^k A_{21}^k}{\left(\rho_1^0\alpha_1^k + \rho_2^0\alpha_3^k\right)\left(v_1^k\right)^2},$$
$$\frac{d\alpha_2^k}{dz} = \frac{A_{12}^k}{\rho_2^0 v_2^k} - \frac{A_{22}^k}{\left(v_2^k\right)^2\rho_2^0}, \quad \frac{d\alpha_3^k}{dz} = \frac{A_{13}^k}{\rho_2^0 v_1^k} - \frac{\alpha_3^k A_{21}^k}{\left(\rho_1^0\alpha_1^k + \rho_2^0\alpha_3^k\right)\left(v_1^k\right)^2}, \tag{2.4}$$

where

$$A_{11}^k = a_{111}^k + a_{112}^k\frac{dp}{dz} + \sum_{l=1}^N I^{l,k}a_{113}^{l,k}; \quad a_{111}^k = Q^k/l; \quad a_{113}^{l,k} = \frac{1}{S^k}\frac{\alpha_1^l + \alpha_1^k}{2}\rho_1^0; \tag{2.5}$$
$$a_{112}^k = -\frac{1}{l}\left(\rho_2^0 v_2^k\alpha_2^k C_2 + \rho_1^0 v_1^k\alpha_1^k C_1 + \rho_2^0 v_1^k\alpha_3^k C_2\right)\frac{dT_s}{dp};$$
$$A_{21}^k = a_{211}^k + a_{212}^k\frac{dp}{dz} + \sum_{l=1}^N I^{l,k}a_{213}^{l,k}; \quad a_{211}^k = \left(Q^k/l + j_{23}^k\right)\left(v_2^k - v_1^k\right) - \frac{f_{21}^k}{S^k} +$$
$$+ \left(\alpha_1^k\rho_1^0 + \alpha_2^k\rho_2^0\right)g + \frac{1}{S^k}\sum_{l=1}^N k_t^{l,k}\left(v_1^l - v_1^k\right)\left[\frac{\rho_1^0}{2}\left(\frac{\alpha_1^k}{\alpha_1^k + \alpha_3^k} + \frac{\alpha_1^l}{\alpha_1^l + \alpha_3^l}\right) + \right.$$

$$+k_z\frac{\rho_2^0}{2}\left(\frac{\alpha_3^k}{\alpha_1^k+\alpha_3^k}+\frac{\alpha_3^l}{\alpha_1^l+\alpha_3^l}\right)\Bigg];\quad a_{212}^k=-\left(\alpha_1^k+\alpha_3^k\right)-\frac{(v_2^k-v_1^k)\rho_2^0\alpha_2^k v_2^k C_2}{l}\frac{dT_s}{dp};$$

$$a_{213}^{l,k}=\frac{1}{S^k}\frac{v_1^l-v_1^k}{2}\left(\frac{\alpha_1^l+\alpha_1^k}{2}\rho_1^0+k_z\frac{\alpha_3^l+\alpha_3^k}{2}\rho_2^0\right);\quad A_{12}^k=a_{121}^k+a_{122}^k\frac{dp}{dz};$$

$$a_{121}^k=-\frac{Q^k}{l}+j_{12}^k-j_{23}^k;\quad a_{122}^k=\frac{\rho_2^0\alpha_2^k v_2^k C_2}{l}\frac{dT_s}{dp};\quad A_{22}^k=a_{221}^k+a_{222}^k\frac{dp}{dz};$$

$$a_{221}^k=j_{32}^k\left(v_1^k-v_2^k\right)-\frac{f_w^k}{S^k}+\frac{f_{21}^k}{S^k}+\alpha_2^k\rho_2^0 g;\quad a_{222}^k=-\alpha_2^k;$$

$$A_{13}^k=a_{131}^k+a_{132}^k\frac{dp}{dz}+\sum_{l=1}^N I^{l,k}a_{133}^{l,k};\; a_{131}^k=j_{23}^k+\frac{1}{S^k}\sum_{l=1}^N k_t^{l,k}k_z\rho_2^0\left(\frac{\alpha_3^l}{\alpha_1^l+\alpha_3^l}-\frac{\alpha_3^k}{\alpha_1^k+\alpha_3^k}\right)+$$

$$+j_{23}^k-j_{32}^k;\quad a_{132}^k=\frac{1}{l}v_1^k\left(\rho_1^0\alpha_1^k C_1+\rho_2^0\alpha_3^k C_2\right)\frac{dT_s}{dp};\quad a_{133}^{l,k}=\frac{1}{S^k}\frac{\alpha_3^l+\alpha_3^k}{2}\rho_2^0 k_z.$$

The necessary N conditions for determining the streams of a volume of gas–drop mixture $I^{l,k}$ (l, k = 1, 2, ..., N) can be obtained if in each cell we sum Eqs. (2.4), (2.5) and take account of the fact that $\sum_{j=1}^{3}\alpha_j^k=1$; then

$$\frac{d\rho_1^0}{dp}\frac{\alpha_1^k}{\rho_1^0}\frac{dp}{dz}-\frac{A_{11}^k}{\rho_1^0 v_1^k}+\frac{\alpha_1^k A_{21}^k}{\left(\rho_1^0\alpha_1^k+\rho_2^0\alpha_3^k\right)\left(v_1^k\right)^2}-\frac{A_{12}^k}{\rho_2^0 v_2^k}+\frac{A_{22}^k}{\left(v_2^k\right)^2\rho_2^0}-\frac{A_{13}^k}{\rho_2^0 v_1^k}+ \\ +\frac{\alpha_3^k A_{21}^k}{\left(v_1^k\right)^2\left(\rho_1^0\alpha_1^k+\rho_2^0\alpha_3^k\right)}=0\quad(k=1,\ldots,N). \tag{2.6}$$

However, the number of unknown streams $I^{l,k}$ (l, k = 1, 2, ..., N) is larger than N, and the system (2.6) in the general case is found to be indeterminate. This indeterminacy arises as a result of the use of assumptions that the pressure is constant in the cross section of the channel. Therefore this approach does not consider small pressure drops between adjacent cells, to which the transverse streams of gas–drop mixture are proportional. The missing relations can be obtained if we take account of the fact that the sum of pressure drops over any closed contour is equal to zero. This, if we assume that the proportionality factors between the pressure drops and the transverse streams are equal to each other, the sum of the streams of gas–drop mixture over any closed contour will also be equal to zero:

$$I^{i_1^m,i_2^m}+I^{i_2^m,i_3^m}+\ldots+I^{i_p^m,i_1^m}=0, \tag{2.7}$$

where the cells i_1^m, i_2^m, ..., i_p^m (m = 1, ..., M) form a closed contour; M is the number of independent contours. Using a well-known theorem from the theory of polyhedra, we can show that the number of cells N plus the number of independent contours M per unit is greater than the number of streams B_p between adjacent cells ($M + N = B_p + 1$). Then Eqs. (2.6), (2.7) form a system of linear algebraic equations for determining the B_p convective streams and the pressure gradient along the channel. The solution of the system (2.6), (2.7) exists and is unique; it can be determined by standard methods. It should be noted that the corresponding systems of equations for determining the transverse convective streams obtained in most other studies [9] are highly nonlinear, which makes it more difficult to prove the existence and uniqueness of the solution and complicates the procedure for determining the streams, especially in channels with a large number of rods.

As a rule, what is fed into the inlet of channels with bundles of heated rods is a liquid which has not been heated to the saturation temperature. Therefore what moves in the initial segment of the channels is insufficiently heated liquid, and then a bubbling vapor–liquid mixture, passing into a dispersed-film regime of flow. The flow of insufficiently heated liquid and bubbling mixture has been described within the framework of a homogeneous model [9]. In carrying out the calculations it was assumed that in each cell the bubbling regime of flow changes to a dispersed-film regime at $\alpha_1^k = \alpha_1^*$, where $\alpha_1^* \simeq 0.75$ is the critical value of the volumetric vapor content obtained in circular pipes [12].

3. Equation of State of the Phases. Mass and Force Interaction between the Components of the Mixture. For closure of the system (2.1)-(2.3) it is necessary to specify the equations of state for the three components of the mixture and to determine the mass and force interaction between them within a cell and the turbulent exchange of gas–drop mixture between adjacent cells. In what follows, we shall investigate the flow of a steam–water mixture and use the following equations of state for water and steam on the saturation line:

$$\rho_2^0 = \rho_3^0 = \text{const}, \quad \rho_1^0 = \frac{1}{\dfrac{1}{\rho_2^0} + \dfrac{l}{T}\dfrac{dT_s}{dp}}, \quad u_2 = u_{20} + C_2(T_2 - T_0),$$
$$T_s = \frac{T^*}{\ln\left(\dfrac{p^*}{p}\right)}, \quad u_1 = u_{2s} + C_2(T_s(p) - T_0) + l(p), \tag{3.1}$$
$$l(p) = l_0 \exp\left[\left(a + b\ln\frac{p}{p_0}\right)\ln\frac{p}{p_0}\right],$$

where T_0, T^*, p^*, l_0, p_0, a, and b are constants selected from the condition of best agreement of (3.1) with the tabulated data. It should be noted that some relations in (3.1) (incompressibility of the liquid, form of the equations for the internal energies of the phases) have already been used in deriving Eqs. (2.3), (2.4).

In order to determine the turbulent exchange between cells, we used a functional relationship [13] which satisfactorily describes the experimental data. For the particular manner in which we subdivided the channel into cells, the functional relationship of [13] for the coefficient of turbulent exchange has the form

$$k_t^{l,k} = \frac{LR_1^{l,k}}{H^{l,k}}\, 0.0128 k_z \Delta v^{l,k} (\mathrm{Re}_1^t)^{-0.125}, \quad \mathrm{Re}_1^t = \frac{\rho_{13}\Delta v^{l,k} M^k}{\mu_1},$$
$$\Delta v^{l,k} = \left|\frac{(v_1^k - v_2^k) + (v_1^l - v_2^l)}{2}\right|, \quad M^k = \sqrt{\frac{S^k}{\pi} + R^2 - R - \delta^k}, \tag{3.2}$$

where M_k is the characteristic dimension of the core of the stream; R is the radius of the rod in the k-th cell; δ^k is the thickness of the liquid film on the k-th rod; $L^{l,k}$ is the length of the boundary between the k-th and l-th cells; $R_1^{l,k}$ is the minimum distance from the surface of the l-th rod to the surface of the k-th rod; $H^{l,k}$ is the distance between the centers of the l-th and k-th rods; μ_1 is the coefficient of dynamic viscosity of the gaseous phase; ρ_{13} is the effective density of the turbulently mixed gas-drop mixture. The quantity ρ_{13} can be conveniently represented in the form

$$\rho_{13} = \frac{\alpha_1^k \rho_1^0 + k_l \rho_2^0 \alpha_3^k}{\alpha_1^k + k_l \alpha_3^k},$$

where the coefficient k_l is introduced for the purpose of qualitatively taking account of the effect of slippage of the drops in turbulent motion of the gas–drop mixture ($k_l = 1$ means complete entrainment of the drops by the gaseous phase, $k_l = 0$ means no entrainment).

It should be noted that in the model under consideration for the flow of a dispersed-film stream, the effect of the turbulent intercell exchange is considerably less than in the homogeneous model. The reason for this is that much of the liquid in the mixture flows along the rods in the form of a film, and for the type of subdivision adopted in subdividing the cross section of the channel into cells, it does not participate in the intercell mass exchange.

The flow of gas–liquid mixture within a cell is determined in large measure by the intensities of the entrainment of the drops from the surface of the liquid film, j_{23}^k, and of their deposition onto the surface, j_{32}^k. The reason that all previously developed models of dispersed-film gas–liquid streams have been limited in nature is that there are no reliable functions known for the intensities of entrainment and deposition. In order to determine these functions, we had to formulate a special series of experimental investigations [8-10]. The results of these investigations over a wide range of variation of the operational parameters (p = 1-12 MPa, v_1 = 5-120 m/sec, $\alpha_3 = 10^{-3}$-10^{-1}) can be generalized by the functional relations given below.

For the intensity of deposition of drops on the liquid film, j_{32}^k, the following expressions were obtained in [9]:

$$j_{32}^k = \frac{\alpha_3^k \rho_2^0 v_1^k}{2(R+\delta^k)} 3.2\cdot 10^{-2} (\alpha_3^k)^{-0.16} (\mathrm{Re}_1)^{-0.12} f(\Pi),$$

$$f(\Pi) = \Pi \text{ for } \Pi > 1, \; f(\Pi) = \Pi^{1/2} \text{ for } \Pi < 1,$$

$$\Pi = 0.16 \frac{\sigma_2}{v_1^k \sqrt{\mu_1 \mu_2}} \left(\frac{\rho_1^0}{\rho_2^0}\right)^{0.26}, \quad \mathrm{Re}_1 = \frac{v_1^k M^k \rho_1^0}{\mu_1},$$

where σ_2 and μ_2 are the coefficients of surface tension and dynamic viscosity of the liquid. The intensity of entrainment of drops from the surface of the film can be represented in the form

$$j_{23}^k = (j_{23}^d)^k + (j_{23}^c)^k + (j_{23}^b)^k,$$

where $(j_{23}^d)^k$ is the intensity of entrainment of the drops that is due to the dynamic action of the stream of vapor on the film; $(j_{23}^c)^k$ is the intensity of secondary entrainment of the drops due to the drops that are knocked out or that fall; $(j_{23}^b)^k$ is the intensity of entrainment of drops that is due to bubble-type boiling in the film. For these terms, as a result* of the experiments we conducted, the following relations were obtained:

$$(j_{23}^d)^k = 0 \quad \text{for} \quad \mathrm{Re}_2 \leqslant 300 \text{ and } r\frac{f_{12}^k}{\pi\sigma_2} N \leqslant 8.5\cdot 10^{-3},$$

$$(j_{23}^d)^k = 0 \quad \text{for} \quad \mathrm{Re}_2 > 300 \text{ and } r\frac{f_{12}^k}{\pi\sigma_2} N \leqslant 2.8\cdot 10^{-5}\,\mathrm{Re}_2,$$

$$\mathrm{Re}_2 = \frac{\rho_2^0 v_2^k \delta^k}{\mu_2}, \quad N = \frac{\mu_1}{\mu_2}\left(\frac{\rho_2^0}{\rho_1^0}\right)^{1/2}, \quad r = \frac{\delta^k}{R+\delta^k};$$

$$(j_{23}^d)^k = \frac{\alpha_2^k \rho_2^0 v_2^k}{2(R+\delta^k)} 29 \frac{r f_{12}^k}{\pi\sigma_2} \frac{1}{\mathrm{Re}_2}\left(\frac{\rho_2^0}{\rho_1^0}\right)^{1/2} r \quad \text{in other cases,} \quad (j_{23}^c)^k = k j_{32}^k,$$

$$k = 1.35\left(\frac{\rho_2^0}{\rho_1^0}\right)^{-0.16} \quad \text{for} \quad \frac{v_1^k \delta^k \rho_1^0}{\mu_1} > A, \quad k = 0 \quad \text{for} \quad \frac{v_1^k \delta^k \rho_1^0}{\mu_1} \leqslant A,$$

$$A = 20\left(\frac{\rho_2^0}{\rho_1^0}\right)^{-0.28} \mathrm{Re}_2^{0.7},$$

$$(j_{23}^b)^k = \frac{\alpha_2^k \rho_2^0 v_2^k}{2\pi(R+\delta^k)} 1.5\cdot 10^{-4} \frac{\mu_1 v_1^*}{\mu_2 \sigma_2}\left(\frac{Q^k S^k}{2\pi R} - 1.5\cdot 10^5\right)^{2.5},$$

$$v_1^* = 119.3(0.71 - p/p_{cr})\,\mathrm{Re}_2^{-0.33} \quad \text{for} \quad \mathrm{Re}_2 \leqslant 2000,$$

$$v_1^* = 9.4(0.71 - p/p_{cr}) \quad \text{for} \quad \mathrm{Re}_2 > 2000,$$

$$p_{cr} = 21.84 \text{ MPa}.$$

The frictional force between the film and the rod, f_w^k, can be represented in the form

$$f_w^k = c_w^k \frac{\rho_2^0 (v_2^k)^2}{2} 2\pi R,$$

where c_w^k is the coefficient of friction, which depends on the flow regime of the film. In [1, 14] the following expressions were obtained:

$$c_w^k = \frac{4}{\mathrm{Re}_2} \quad \text{for} \quad \mathrm{Re}_2 \leqslant 400, \quad c_w^k = \frac{0.057}{\mathrm{Re}_2^{0.25}} \quad \text{for} \quad \mathrm{Re}_2 > 400.$$

The expression for the force interaction between the core of the stream and the liquid film can be represented in the form

*The experimental data were processed by B. I. Nigmatulin and V. E. Nikolaev.

$$f_{12}^k = c_{12}^k \frac{\rho_1^0 (v_1^k - v_2'^k)^2}{2} 2\pi (R + \delta^k),$$

where c_{12}^k is the coefficient of friction between the gas—drop core and the liquid film; v'_2 is the velocity of the liquid at the surface of the liquid film ($v'_2 \approx 1.14 v_2$ from [15]). The variation of c_{12}^k was found in [8] on the basis of experiments in circular pipes. It cannot be used directly in calculations for channels with bundles of rods. The reason for this is that the expression for c_{12}^k includes the average-flow-rate thickness of the film referred to the radius of the pipes. In the method of subdivision of the cross section of the channel into cells that we have adopted, the average-flow-rate thickness of the film is more conveniently referred to M^k, the average distance from the surface of the film to the boundaries of the cell (3.2). By analogy with [8], we shall use the following expression for c_{12}^k:

$$c_{12}^k = c\left(1 + B\left(\delta^k / M^k\right)^{1.3}\right),$$

where B is a constant; c = 0.008. It should be noted that the proposed model for calculating the gas—liquid dispersed-film streams in channels with bundles of rods contains a single undetermined parameter, B.

4. Results of Some Calculations. In this section we give the fundamental results of calculations for the hydrodynamic resistance and the distribution of the liquid over the cross section and along the length of the channel as functions of the operational parameters of the mixture. The results of the calculations for the heat-exchange crisis when dispersed-film steam—water streams flow in channels with bundles of rods will be discussed in a separate study by the authors.

We determine the parameter B from the condition that the calculated and experimental values of the pressure drop due to friction in one of the regimes obtained in [16] must coincide. The experiments in [16] were carried out in an unheated channel with a bundle of seven rods (D = 13 mm, H^{kl} = 15 mm) and an external jacket with an inner diameter D_s = 55 mm, simulating an infinite number of rods. This reduced the hydrodynamic nonequivalence of the individual cells, and correspondingly, there was a decrease in the effect of the intercell streams on the hydrodynamic resistance of the channel. Therefore the experiments in [16] were most convenient for determining the values of the parameter B. A value of B = 150 was chosen for the regime with p = 4 MPa, specific mass flow rate of the mixture G = 2600 kg/m²·sec, and mass flow-rate steam content x_1 = 0.55. This made it possible to describe satisfactorily all the experimental data of the various authors with whose work the comparison was made.

It should be noted that the resulting value of B differs by only 25% from the corresponding value of this constant in the case of circular pipes [8] (in circular pipes B = 210). A value of the constant B close to 150 can be obtained analytically if we assume that each element of the liquid film interacts with the gas—drop stream in the same way as in a pipe, while the total frictional force and the intensity of the mass interaction are obtained by integration of the local intensities on the surface of the film.

Figure 2 shows a comparison of the calculated (solid curves represent calculation by the present method, dashed curves represent calculation by the formula from [3]; curves 1–3 correspond to G = 870, 1200, and 2600 kg/m²·sec) and experimental [16] (the points ▲, ●, × correspond to G = 870, 1200, 2600 kg/m²·sec) data on the pressure drop due to friction at p = 4 MPa, depending on the channel-average mass flow-rate vapor content for various values of specific mass flow rate of the mixture. The formula of [3] yields results which diverge very seriously from the experimental results when the flow-rate vapor content values are small and the specific flow rates are large. It can be seen from Fig. 2 that within the framework of this model it is possible to describe the nonlinear character of the variation of $\Delta p_{fr}/L$ as a function of x_1 and to take account correctly of the variation of $\Delta p_{fr}/L$ as a function of G. It should be noted that in order to describe the indicated stratification within the framework of a homogeneous model, we must introduce an empirical function which takes account of the different inhomogeneity of the stream for different values of G.

Figures 3 and 4 show the variation of the main hydrodynamic characteristics of a steam—water dispersed-annular stream which determine the integral properties of the flow (hydraulic resistance, heat-exchange crisis) as functions of the operational parameters in a channel with

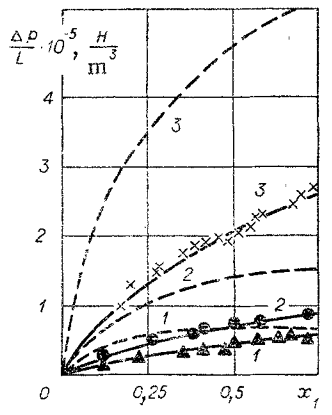

Fig. 2

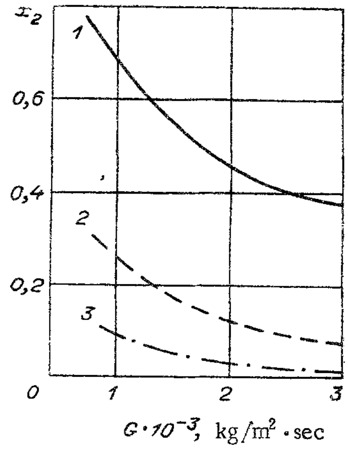

Fig. 3

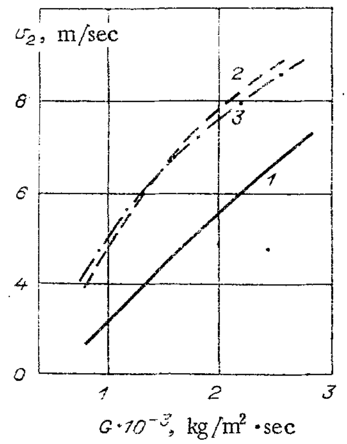

Fig. 4

a bundle of rods [16]. In this channel the cells were hydrodynamically equivalent, and therefore in Figs. 3 and 4 we show the characteristics of the stream in the central cell.

Figure 3 shows the variation of the relative flow rate of the liquid in the film, x_2, equal to the ratio of the liquid flow rate in the film to the total mixture flow rate in the cell, as a function of G and x_1 for p = 4 MPa (curve 1 corresponds to $x_1 = 0.1$, 2 to $x_1 = 0.4$, 3 to $x_1 = 0.7$). Figure 3 shows the values of x_2 for conditions of hydrodynamic equilibrium, i.e., for the case when the distribution of the liquid in the cells no longer depends on the length of the channel. In Fig. 3 we can see a decrease in x_2 as G and x_1 increase. This is explainable by the fact that the intensity of entrainment of the drops from the surface of the film depends strongly on the velocity of the gaseous phase. As was shown in [1], in the flow of a dispersed-annular steam-water stream in circular pipes, a decrease in x_2 leads to a decrease in the force interaction between the core of the stream and the film and makes it possible to explain the mechanism of the hydraulic-resistance crisis.

The calculations show that for fixed specific flow rate, when the pressure increases, the values of x_2 and δ (the average-flow-rate thickness of the film) will increase, and the value of δ for large values of x_1 ($x_1 \gtrsim 0.7$) will vary only slightly.

Figure 4 shows the variation of the average-flow-rate velocities of the film as functions of G for a pressure of p = 4 MPa and various values of x_1 (for curve 1, $x_1 = 0.1$; curve 2, $x_1 = 0.4$; for curve 3, $x_1 = 0.7$). It can be seen that the velocity of the film increases with increasing G and x_1, but for large values of the mass flow-rate vapor content the film velocity will depend only slightly on it. The reason for this is that as x_1 increases, there is an increase in the velocity of the gas-drop core of the stream but the average-flow-rate thickness of the film decreases.

In channels with bundles of rods of considerable length (of the order of several meters) it is customary to set up spacing networks along the length, with a specified interval. The

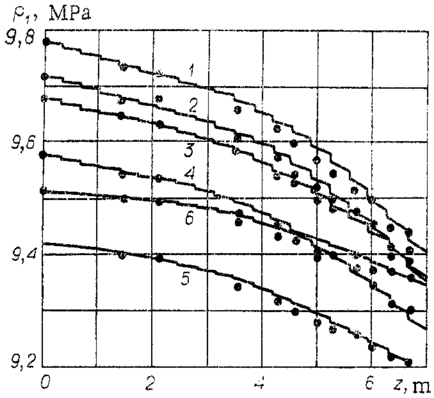

Fig. 5

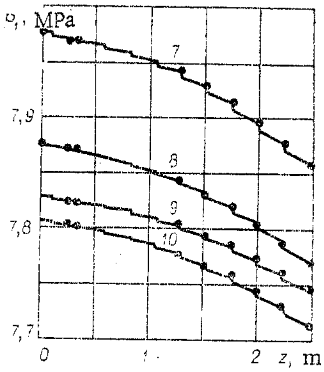

Fig. 6

TABLE 1

No. regime	p_0, MPa	G, kg/m² · sec	Δt, °C	Q, MW	No. regime	p_0, MPa	G, kg/m² · sec	Δt, °C	Q, MW
1	9,78	3106	48	4,39	6	9,515	1469	42	3,02
2	9,73	2985	47	4,27	7	7,98	2227	38	3,53
3	9,68	2239	45	4,27	8	7,88	2027	38	3,38
4	9,58	2468	46	4,05	9	7,83	1565	39	3,0
5	9,42	1721	48	3,7	10	7,81	1815	36	3,14

pressure drop across each spacing network can be determined from the standard function

$$\Delta p = \xi \frac{\rho v_1^2}{2},$$

where ξ is the coefficient of resistance of the spacing network, which depends on its specific geometry.

Figures 5 and 6 show the resistance of calculated and experimental pressure diagrams [3] along channels with 19 and 37 heated rods, respectively. In the first channel the outer diameter of the rods was 13.5 mm, the arrangement of the rods in the bundle is shown in Fig. 1, and the inner diameter of each jacket was 80 mm. In the second channel the outer diameter of each rod was 9 mm, the rods were arranged at the vertices of an equilateral triangle with a side of 12.2 mm, and the width across flats of the hexagonal jacket was 77 mm. Table 1 shows the initial data for the calculated regimes. Here G denotes the specific mass flow rate of the mixture averaged over the cross section of the channel; Δt is the underheating of the water at the channel inlet; Q is the total thermal power generated in the channel. The numbers of the curves in Figs. 5 and 6 correspond to the numbers of the regimes in Table 1, and the points indicate the experimental values of the pressure. The regimes in the first channel were calculated with a value of ξ selected for regime 1, and in the second with ξ values selected for regime 7.

The calculated pressure curves shown in Figs. 5 and 6 are in good agreement with the experimental curves.

Figure 7 shows the distributions along the channel of the liquid-film irrigation density of the rods, Γ_2, in various cells, and also of the outer jacket (curves 2-4 were constructed for the second rod from the center, the third rod from the center, and the outer jacket). Here the term "irrigation density" means the ratio of the liquid flow rate in the film to the wetted perimeter of the rod or the jacket in the cell. It can be seen that at the outlet from the channel much of the liquid flows along the unheated jacket.

A feature of great interest is the length of the initial segment of the unheated channel, in which the parameters of the dispersed-film stream take on their hydrodynamic-equilibrium values. We can consider two limiting methods for introducing liquids into the channel: all

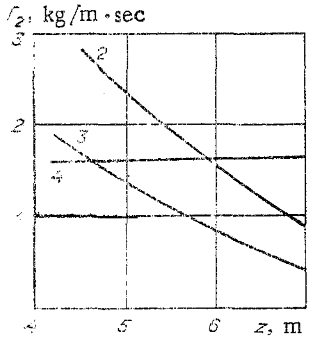

Fig. 7

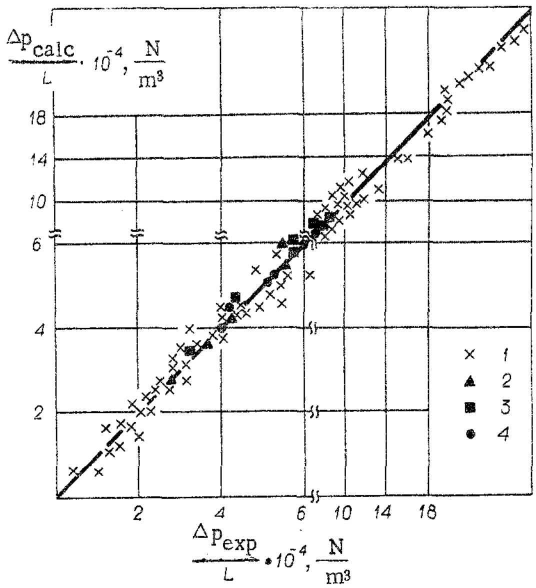

Fig. 8

of the liquid in the form of drops in the core of the stream, or all of the liquid in films. Calculations show that the length of the segment in which the dispersed-film stream goes into equilibrium depends substantially on the initial distribution of the liquid in the cells. For example, when G = 2700 kg/m²·sec, for a 19-rod bundle this length is ∿0.8 m in the first case and ∿2 m in the second.

Figure 8 shows a comparison of the experimental pressure drops due to friction, $\Delta p_{exp}/L$, with the calculated pressure drops, $\Delta p_{calc}/L$, in channels with bundles of 7, 19, and 37 rods (the points numbered 1 are experiments from [16], those numbered 2 from [17], those numbered 3 from the first channel in [3], and those numbered 4 from the second channel in [3]). It can be seen that within the framework of the proposed model the values of $\Delta p_{exp}/L$ can be described essentially to within ±10%.

The above comparison of experimental and calculated characteristics of dispersed-film streams in a channel with bundles of rods having different geometries shows that the model we have worked out describes adequately the fundamental hydrodynamic characteristics of such streams.

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SECONDARY FLOWS IN AN UNSTABLE BOUNDARY LAYER

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Analysis of a large amount of experimental data on the structure of the transition region from laminar to turbulent flow on a planar plate [1, 2] makes it possible to draw the conclusion that, following the stages of linear development of the original instability and weak nonlinear development of perturbations, their three-dimensional growth takes place, and at some moment the evolution of wave motion is inevitably three-dimensional. Experiments determine the existence of long-wave eddy formation with the axis along the main flow direction, as a result of which there is a redistribution of mean flow momentum and the appearance of a secondary three-dimensional regime. The experimentally observed longitudinal vortices are periodic in the coordinate z, as is schematically illustrated in Fig. 1. Significant progress has been achieved in the study of the nonlinear transition stages, which cannot be said on analysis of three-dimensional effects as applied to the mean flow.

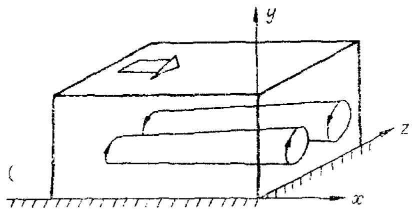

Fig. 1

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